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Combining Market and Accounting-based Models for Credit Scoring Using a Classification Scheme Based on Support Vector Machines

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COMBINING MARKET AND ACCOUNTING-BASED MODELS FOR CREDIT SCORING USING A CLASSIFICATION SCHEME BASED ON SUPPORT VECTOR MACHINES

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Abstract

Credit risk rating is a very important issue for both banks and companies, especially in periods of economic recession. There are many different approaches and methods which have been developed over the years. The aim of this paper is to create a credit risk rating model combining the option-based approach of Black, Scholes, and Merton with an accounting-based approach which uses financial ratios. While the market model is well-suited for listed firms, the proposed approach illustrates that it can also be useful for non-listed ones. In particular, the option-based model is implemented to a group of listed firms and its results are applied in order to develop a model for credit risk evaluation of non-listed firms, using financial ratios. This approach is tested on a sample of Greek firms and the results are compared to other already established models.

Keywords: Credit risk, Black-Scholes-Merton model, Credit rating, Support vector machines

1. INTRODUCTION

Credit risk refers to the probability that a client will not be able to meet his/her debt obligations (default). Over the years, many factors have contributed to the increasing

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importance of accurate credit risk measurement. Altman and Saunders (1997) list five main issues, which are still valid in the current context: (i) a worldwide structural increase in the number of defaults, (ii) a trend towards disintermediation by the highest quality and largest borrowers, (iii) more competitive margins on loans, (iv) a declining value of real assets (and thus collateral) in many markets, and (v) a dramatic growth of off-balance sheet instruments with inherent default risk exposure including credit risk derivatives. Credit risk measurement is nowadays a critical issue as demonstrated by the outbreak of the credit crisis.

In a credit risk management context, the accurate estimation of the probability of default is a crucial point. Credit rating models (CRMs) are widely used for that purpose. CRMs evaluate the creditworthiness of a client, estimate the probabilities of default, and classify the clients into risk groups. The accounting-based credit scoring approach is probably the most widely used one. In a corporate credit granting context, credit scoring models combine key financial (accounting) and non-financial data into an aggregate index indicating the credit risk of the firms. Credit scoring models can be constructed with a variety of statistical, data mining, and operations research techniques (e.g., logistic regression, neural networks, support vector machines, rule induction algorithms, multicriteria decision making, etc.). Comprehensive reviews of this line of research can be found in Thomas (2000), Papageorgiou et al. (2008), and Abdou and Pointon (2011). Despite their success and popularity, traditional credit scoring models are mostly static and they are based on historical accounting data which describe the current and past performance of a firm but may fail to represent adequately the future of the firms and the trends in the business environment (Altman and Saunders, 1997; Agarwal and Taffler, 2008). This is particularly important in the context of an economic turmoil, where exogenous conditions deteriorate significantly in a short time period, thus affecting corporate activity and leading to increased credit risk levels throughout the market. Mensah (1984) and Hillegeist et al. (2004) also discuss issues related to the accounting standards and practices, which affect the quality of the information that financial statements provide, as well as the discrepancies between book and market values.

The shortcomings of accounting-based credit scoring models have led to the consideration of a wide variety of alternative approaches (comprehensive overviews can be found in Altman and Saunders 1997; Altman et al., 2004). Among them, structural models have attracted considerable interest. Structural models are based on the contingent claims approach (Black and Scholes, 1973; Merton, 1974) and use market information to assess the probability of default. In efficient markets, stock prices reflect all the information related to the current status of the firms as well as expectations regarding their future progress (Agarwal and Taffler, 2008). Furthermore, market data are constantly updated as the investors and market participants take into consideration update information relevant to the performance of a firm and the conditions prevailing in its operating environment. These features of market data and models indicate that they may be better suited for default prediction and credit risk measurement. Actually, several studies provide empirical results in support of market models in the context of credit risk modeling and bankruptcy prediction (Hillegeist et al., 2004; Agarwal and Taffler, 2008). Market models have also been shown to contribute in the construction of improved hybrid systems in combination with accounting-based models (Li and Miu, 2010; Yeh et al., 2012).

Despite their strong theoretical grounds and good predictive power, market models are limited to listed firms. Therefore, their extension to private non-listed firms has attracted some interest over the past decade. Moody's KMV RiskCalc™ model (Dwyer et al., 2004) is a commercial implementation, which has been employed in several countries with positive results (Blochwitz et al., 2000; Syversten, 2004). Altman et al. (2011) used US data to examine the potential of developing multivariate regression models providing estimates for the probability of default implied by a market model. The authors found that this approach provides similar results to default prediction models, thus concluding that both approaches should be treated as complementary sources of information.

This study extends the results of Altman et al. (2011) by investigating the applicability of a market-based credit risk modeling approach in a context where the hypotheses of market efficiency may by invalid (Majumder, 2006). In particular, we test whether a definition of default on the basis of a market model can be employed to build a credit scoring model for non-listed firms and compare the results to a default prediction model fitted on historical default data. The analysis is based on data from Greece over the period 2005–2010 using samples of listed and non-listed firms. The Greek case provides a challenging context due to two main reasons. First, the Greek stock market, after flourishing at the end of the 1990s, it entered a period characterized by increasing volatility, decreasing liquidity, and high market concentration with few large capitalization stocks dominating the market. These features became even clearer during the international credit crisis and the subsequent sovereign debt crisis that hit the country, thus putting into serious question the efficiency of the Greek stock market (Dicle and Levendis, 2011). Second, the crisis had a particularly strong effect on the Greek economy, with a sharp deterioration of the general economic and business conditions, which led to an unprecedented increase in the number of defaults and bankruptcies over a very short period of time. Thus, credit risk management becomes a challenging issue in this context, and the peculiarities of the Greece case cast doubts on whether an approach based on the grounds of a market model could actually provide useful results.

On the methodological side, instead of employing a statistical linear regression approach, non-parametric machine learning techniques are employed based on the framework of support vector machines (SVMs). In particular, the analysis is performed in two stages. First, the basic model introduced by Black and Scholes (1973) and Merton (1974), is employed to assess the probabilities of default for listed firms (henceforth referred to as the BSM model). The listed firms are classified into risk groups under different risk-taking scenarios. Risk assessment and classification models are then developed using linear and nonlinear support vector machines (SVMs), as well as a recently developed innovative additive SVM model that suits well the requirements of credit risk rating. Logistic regression is also employed for comparative purposes and feature selection. The obtained models are then applied to a sample of non-listed firms. The comparison against traditional credit scoring models fitted on historical default data shows that the market-based modeling approach provides very competitive results.

The rest of article is organized as follows. Section 2 provides a brief reminder of the BSM model and presents the SVM classification approach employed in the analysis. Section 3 is devoted to the empirical analysis, including the presentation of the data and the obtained results. Finally, section 4 concludes the paper, summarizes the main findings of this research, and proposes some future research directions.

2. METHODOLOGY

2.1. The Market Model

The introduction of the BSM model through the works of Black and Scholes (1973) as well as Merton (1974), led to the development of the research on structural models for credit risk modeling. In the BSM framework, a firm is assumed to have a simple debt structure, consisting of a single liability with face value L maturing at time T. The firm defaults on its debt at time T, if its assets' market value is lower than L. In this context, the firms' market value of equity (E) is modeled as a call option on the underlying assets (A). The value of the equity is given by the Black-Scholes formula for option pricing:

$$E = A\Phi(d_1) - Le^{-rT}\Phi(d_2) \tag{1}$$

with

$$d_1 = \frac{\ln(A/L) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

where r is the risk-free rate, σ is the volatility of the asset returns, and $\Phi(\cdot)$ represents the cumulative normal distribution function.

Furthermore, under the Metron's assumption that equity is a function of assets and time, the following equation is derived from Itô's lemma (Hull, 2011):

$$\sigma_E = \frac{\sigma A \Phi(d_1)}{E} \tag{2}$$

Equations (1) and (2) can be solved simultaneously with analytic or iterative procedures (Hillegeist, 2004; Vassalou and Xing, 2004) to estimate the market value of assets (A) and the volatility of assets' return σ. Then the probability of default (PD) at time T is defined by the probability that the market value of assets at time T is below the default point L (face value of debt) is:

$$PD = \Phi\left(-\frac{\ln\frac{A}{L} + (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}} \right) \tag{3}$$

where μ is the expected return on assets, which can be estimated from the annual changes in A obtained from the solution of equations (1) and (2).

In the context of the basic BSM model, several variants have been introduced in the literature (see Agarwal and Taffler, 2008 for a comparative analysis). In this study we employ the approach of Bharath and Shumway (2008), who proposed a very simple variant, under which the market value of assets is set equal to equity and the liabilities (i.e., $A = E + L$) and the volatility parameter is approximated by:

$$\sigma = \frac{E}{A}\sigma_E + \frac{E}{A}(0.05 + 0.25\sigma_E) \tag{4}$$

Bharath and Shumway (2008) suggest setting μ equal to the annualized equity returns (r_E), but in this study we instead set $\mu = \max\{r_E, r\}$ in accordance with the arguments developed by Hillegeist (2004). Furthermore the time period T is set equal to one year (as default prediction models are usually developed to provide one-year ahead estimates), the firm's equity E is taken from the market capitalization of the firms, and L is defined following an approach similar to the one of the Moody's KMV model (Dwyer et al., 2004), using the book value of short term liabilities plus half of the long term debt.

Despite its simplicity (as it does not require the solution of the system of equations (1)-(2)), the results of Agarwal and Taffler (2008) have shown that this simple the introduced by Bharath and Shumway (2008) performs remarkably well, even outperforming approaches based on the traditional BSM model.

2.2. Extrapolation to Non-listed Firms

The BSM model described in the previous section is only applicable to listed firms as it is based on market data. In order to employ the model for non-listed firms, a set of data available for both listed and non-listed firms should be used to construct a model that will provide estimates on the probability of default, similar to the ones obtained with the BSM approach.

To implement the above process we adopt a classification modeling approach, assuming that on the basis of the results of the market model (estimated probabilities of default, PDs), a listed firm can be classified into one of predefined default risk groups (e.g., low, medium, or high risk). In the simplest dichotomous setting two risk groups can be considered corresponding to high and low risk cases. This approach is in accordance with the common approach adopted for the development of credit scoring and rating systems on the basis of historical default data. The classification of the listed firms in the predefined groups can be easily performed by introducing a threshold on the PDs estimated through the market model. Firms with PD higher than the selected threshold are classified as high risk, otherwise they are assigned to the low risk group. The PD threshold can be specified considering the risk-taking policy of particular credit risk managers and bearing in mind the general conditions prevailing in the economy of a country.

On the basis of the credit risk classification of the listed firms, a number of methods can be used to build a model that combines a set of attributes and provides recommendations on the credit risk level of the firms. In this study, the support vector machines (SVMs) modeling approach is employed. SVMs have become an increasingly popular statistical learning methodology for developing classification models (Vapnik, 1998) with many successful applications in financial decision-making problems, including credit scoring (see for instance the recent studies of Martens et al., 2007; Bellotti and Crook, 2009; Huang, 2011; Su and Chen, 2011).

In a dichotomous credit risk modeling setting, a set of m training observations $\{\mathbf{x}_i, y_i\}_{i=1}^m$ is available corresponding to firms in default ($y_i = 1$) or non-default ($y_i = -1$). Each observation of a firm's data is a multivariate vector $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})$ described over n default predictor attributes.

In the simplest case, a linear classifier $F(\mathbf{x}) = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n$ can be assumed. With such a model, a firm is classified as non-default if $F(\mathbf{x}) > 0$, otherwise it is assigned into the default group. The classifier that discriminates the two groups in an optimal manner can be constructed through the solution of the following quadratic optimization problem:

$$\begin{aligned} &\underset{\mathbf{s}\geq\mathbf{0},\, \boldsymbol{\beta},\alpha\in\mathbb{R}}{\text{minimize}} && \|\boldsymbol{\beta}\|^2 + C\mathbf{e}^\top\mathbf{s} \\ &\text{subject to}: && \mathbf{Y}(\mathbf{X}\boldsymbol{\beta} + \alpha) + \mathbf{s} \geq \mathbf{e} \end{aligned} \tag{5}$$

where $\mathbf{Y}$ is a $m \times m$ diagonal matrix with the class labels in its diagonal (1 for the non-defaulted cases in -1 for the defaulted ones), $\mathbf{X}$ is the $m \times n$ matrix with the training data, $\mathbf{e}$ is a vector of ones, $\mathbf{s}$ is a vector of non-negative slack variables associated with the misclassification of the training observations, and $C > 0$ is a user-defined parameter representing the trade-off between the total classification error and the regularization term $\|\boldsymbol{\beta}\|^2$.

Nonlinear decision models can also be developed by mapping the problem data to a higher dimensional space (feature space) through a transformation imposed implicitly by a symmetric kernel function $K(\mathbf{x}_i, \mathbf{x}_j)$. The nonlinear model has the following form:

$$F(\mathbf{x}) = \alpha + \sum_{i=1}^{m} \lambda_i y_i K(\mathbf{x}_i, \mathbf{x}) \tag{6}$$

where $0 \leq \lambda_1, \ldots, \lambda_m \leq C$ are Lagrange multipliers associated with the training data, which are obtained by solving a dual version of problem (6), after plugging in the kernel function. In this study, the RBF kernel with width $\gamma > 0$ is employed:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\gamma \left\|\mathbf{x}_i - \mathbf{x}_j\right\|^2\right) \tag{7}$$

Except for the traditional linear and nonlinear SVM classifiers, Doumpos et al. (2007) have also developed an SVM-based algorithm for constructing additive decision models of the following form:

$$F(\mathbf{x}) = \alpha + \sum_{k=1}^{n} f_k(x_k) \tag{8}$$

where $f_1, \ldots, f_n$ are (smooth) functional-free attribute functions (partial evaluation functions), which have a functional-free form inferred directly through the training data. Additive models retain the simplicity, transparency, and interpretability of linear models, combined with the nonlinear behavior of more complex classifiers, which is an important feature in the context of credit scoring (Martens et al., 2007; Martens and Baesens, 2010).

The algorithm developed by Doumpos et al. (2007) for training the additive model is based on the combination of multiple linear SVM classifiers fitted on different piecewise linear transformations of the training data obtained by dividing each attribute's domain into proper subintervals. The algorithm has been shown to be computationally effective for large data sets and its classification performance was found to be superior compared against other SVM models in a credit risk rating setting.

3. EMPIRICAL ANALYSIS

3.1. Data and Variables

Two data samples are used in the analysis. The first includes 1,314 firm-year observations involving (non-financial) firms listed in the Athens Stock Exchange (ASE) over the period 2005–2010. For each year t in that period, the sample includes all firms traded throughout year t in ASE and their daily logarithmic returns over the whole year were used to estimate their PDs at the end of year t. The second sample consists of 10,716 firm-year observations for non-listed Greek firms from the commercial sector over the period 2007–2010. The sample of non-listed firms has been obtained from ICAP SA, a leading business provider in Greece. All observations in this sample are classified as default or non-default on the basis of the definition of default employed by ICAP, which considers a range of default-related events such as bankruptcy, protested bills, uncovered cheques, payment orders, etc. Table 1 presents the number of observations per year for both samples.

Insert Table 1 around here

On the basis of data availability and the relevant literature seven financial ratios are used to describe the financial status of the firms in both samples. The selection of the appropriate financial ratios is a challenging issue. In fact, there is a big variety of ratios that can be used as proxies of the same financial dimensions (leverage, liquidity, profitability, etc.). Furthermore, time and cost issues arise when using a large number of ratios and this can also cause multicollinearity problems. Table 2 presents the selected ratios together with their expected relationship (sign) to the creditworthiness of the firms. A positive sign (+) is used to indicate ratios which are positively related to the creditworthiness of the firms, in the sense that higher values in these ratios are expected to improve the creditworthiness of the firms. The rest of the ratios are assigned a negative sign (–) indicating their negative relationship with the performance and viability of the firms (i.e., as these ratios increase the likelihood of default is also expected to increase).

Insert Table 2 around here

The selected ratios cover all major dimensions of a firm's financial performance, including profitability, leverage, solvency, liquidity, and managerial performance (Courtis, 1978; Crouhy et al., 2001). Profitability ratios are used to take into consideration the ability of the firms to generate earnings from their operating activity. Two profitability ratios are used in this study. The gross profit to sales ratio measures the gross profit margin of a firm on the basis of its revenues and cost of sales, whereas the earnings before interest and taxes to total assets ratio measures the return on assets (ROA) of the firms. Financial leverage and solvency indicate the debt burden of the firms and their ability to meet their debt obligations. The ratio of total liabilities to total assets is probably the most popular measure of leverage and it is negatively related to the viability of the firms. On the other hand, the interest expenses to sales ratio, provides an indication of the debt servicing ability of the firms. Liquidity ratios focus on the firms' ability to meet their short-term obligations. In this study two indicators are used, namely the current ratio (current assets/short-term liabilities) and the sales to short-term liabilities ratio, both of which are positively related to the financial performance and viability of the firms. Finally, the accounts receivable turnover ratio (accounts receivable$\times$365/sales) is used as an indicator of the management competence in implementing proper credit policies towards the clients and debtors of a firm.

Table 3 summarizes the averages of the selected financial ratios for both samples (listed and non-listed firms). For the sample of non-listed firms, the averages are also reported for each group of observations (i.e., default, non-default). The comparison between the listed and non-listed firms provides mixed results. Listed firms seem to be less profitable and their interest expenses are higher, but their debt burden is lower, and they follow a tighter credit policy towards their debtors. According to the results of the non-parametric Mann-Whitney test, the differences between the two samples are significant at the 1% level, except for the gross profit to sales ratio. As far as it concerns the differences between the defaulted and non-defaulted firms from the sample of non-listed companies, they are all found significant at the 1% level under the Mann-Whitney test.

Insert Table 3 around here

3.2. Market Model's Estimates

The BSM model was applied to the sample of listed firms with the parameters discussed in section 2.1. Figure 1 presents the estimated probabilities of default (PD), averaged over all firms for each year of the analysis. The results are clearly in accordance with the general economic conditions that emerged after the global credit crisis and the subsequent sovereign debt crisis in Greece. In particular, in 2006 the average PD decreased to its minimum level at 2.9%, but it peaked up significantly in the following years, exceeding 10% in 2009 and further climbing to 17.6% in 2010, when the Greek crisis started to unfold.

Insert Figure 1 around here

Following the approach discussed in section 2.2, the listed firms are classified as low or high risk on the basis of their estimated PDs.[1] In the current analysis we test different probability (PD) thresholds to perform this classification and investigate the robustness of the results under different risk-taking scenarios. It should be noted that using higher values for the PD threshold leads to a decreasing number of high risk firms (i.e., cases with PD above the threshold), thus corresponding to risk-taking credit policies. Figure 2 presents the relationship between the PD threshold and an empirically estimated risk level using the sample of listed firms. The risk level is simply defined as the percentage of observations classified into the high risk group according to the adopted PD threshold. The illustration shows that the risk

[1] Except for the classification scheme a regression approach was also examined, using both regression (OLS) and non-parametric (SVM regression) techniques for the logit of the estimated PDs. The results were found inferior to the classification setting.

level increases exponentially as the PD thresholds decreases. PD thresholds below 10% seem to be too conservative with many cases classified into the high risk group, whereas thresholds above 30% lead to very few high risk classifications (e.g., the empirical risk falls well below 5% for thresholds above 40%).

Insert Figure 2 around here

On the basis of the above results and taking into account the conditions prevailing in Greece during the period of the analysis, we focus on three characteristic risk-taking scenarios. First, under a conservative risk setting corresponding to a larger number of high risk firms, the threshold is set at 10%. On the other hand, in a risk-prone scenario the probability threshold is increased to 30%, whereas under an intermediate risk setting the threshold is set to 20%.

Using these specifications, Table 4 presents some detailed statistics on the number of high and low risk (listed) firms in each year, together with the average PDs estimated from the market model over all cases belonging in each group. In accordance with the results shown in Figure 1, it is clearly evident that under all settings, the number of high risk firms has increased significantly over the period 2007–2010. In 2006 the percentage of firms classified in the high risk class ranged between 8% (for the 30% PD threshold case) up to 15.6% (under the 10% PD threshold scenario). On the other hand, in 2010 the proportion of high risk firms in the sample increased to 58.4%, 42.1%, and 27.6%, respectively under the 10%, 20%, and 30% PD threshold settings. Furthermore, in accordance with the results in Figure 2, it is clearly evident that the number of high risk firms decreases significantly as the PD threshold increases. It is also worth noting that the PDs estimated from the market model are well differentiated between the two groups. The overall average PD for the low risk firms ranges between 1.59–5.41% (depending on the PD threshold setting used to classify the firms), whereas for the high risk firms it ranges between 25.93% and 42.19%. As expected, the PDs for both groups of firms (low and high risk) increase with the PD classification threshold.

Insert Table 4 around here

Table 5 summarizes the averages of the selected financial ratios for each group of cases (low and high risk) defined through the market model's results. It is worth noting that the averages for the low risk firms do not change significantly under the three classification settings (i.e., the different PD thresholds). On the other hand, the average performances of the high risk firms gradually deteriorate as the PD threshold increases. In all cases the differences in the financial characteristics of the two groups of firms are statistically significant at the 1% level (according to the Mann-Whitney test).

Insert Table 5 around here

3.3. Generalization to the Non-listed Firms

On the basis of the market model's classifications described in the previous section, SVM models (linear, RBF, additive) were developed providing recommendations on the credit risk level of the firms on the basis of the selected ratios. The parameters of all models were calibrated using the pattern search procedure proposed by Momma and Bennett (2002) based on 5-fold cross validation. Stepwise logistic regression (LR) is also employed for comparison purposes as well as for attribute selection. LR is the most widely used statistical approach for financial decision making with numerous applications in several financial classification problems, including credit scoring. Furthermore, stepwise LR provides a simple and convenient approach for selecting statistically significant predictor attributes in a multivariate setting. In this study a forward selection procedure was employed at the 5% significance level.

Table 6 presents the financial ratios' coefficients in the models developed through the stepwise LR procedure. The coefficients of the ratios in the linear SVM models are also reported for both the set of ratios selected by LR (SVM-LR) and the full set of ratios (SVM-all). While similar information for the contribution of the predictor attributes is not available for nonlinear SVM RBF models, the additive approach (ASVM) provides such estimates through the examination of the attributes' partial evaluation functions $f_1, \ldots, f_n$ in model (9). In the context of ASVM the relative importance of the financial ratios is measured by the standard deviation of the ratios' partial evaluation functions, normalized so that the contributions of all variables' sum up to one.

Insert Table 6 around here

According to the results of Table 6, the coefficients of all ratios selected by the stepwise LR model have the expected signs, both in the LR and the linear SVM models. On the other hand, in the linear SVM model developed with all attributes, the ratios not selected by the stepwise procedure have coefficients with opposite signs compared to their expected relationship with the probability of default. In the ASVM model developed with all ratios, the ratios current assets to short-term liabilities (CA/STL), return on assets (EBIT/TA), and sales to short-term liabilities (S/STL) are the most important predictors under all settings. In the ASVM models developed with the four ratios selected by LR (ASVM-LR), the solvency ratio (TL/TA) is the most important predictor, followed by return on assets. Figure 3 illustrates the partial evaluation functions (standardized to zero mean and unit variance) of these two ratios in the ASVM models developed with the full set of variables under different PD classification thresholds. It is clearly evident that the firms are evaluated in terms of their ROA through an S-like function, whereas solvency contributes in all models through a decreasing function. It is also worth noting that the form of the evaluation functions is robust to the three different settings (PD thresholds).

Insert Figure 3 around here

In order to evaluate the predictive performance of the models, they were applied to the sample of non-listed firms and their results were compared against the actual credit status of the firms (as described in section 3.1). The performance of the models is analyzed through two popular measures, namely the area under the receiver operating characteristic curve (AUROC, Fawcett, 2006) and the Kolmogorov-Smirnov distance. AUROC provides an overall evaluation of the generalizing performance of a classification model without imposing any assumptions on the misclassification costs or the prior probabilities. AUROC is a popular measure for the evaluation of credit rating models (Sobehart and Keenan, 2001; Blöchlinger and Leippold, 2006). In a credit rating context, and assuming a credit scoring function $F(\cdot)$ defined such that higher values indicate lower probability of default, AUROC represents the probability that a non-defaulted firm will receive a higher credit score compared to a firm in default. A perfect model has AUROC equal to one, whereas a model with AUROC close to 0.5 has no predictive power.

In addition to AUROC, the Kolmogorov-Smirnov (KS) distance is also used as an additional performance measure (Thomas et al., 2002). The Kolmogorov-Smirnov distance is defined as the maximum difference between the cumulative distributions of the credit scoring function F for the two groups of firms (default minus non-default). A large positive difference (i.e., close to one) indicates that the credit scores assigned to the cases in default are concentrated to the lower part of the scoring scale, whereas the scores of the non-defaulted cases are concentrated to higher values of the evaluation scale and consequently the distribution of F is significantly different for the two groups.

Comprehensive results on both evaluation measures for all of the developed models are given in Tables 7 and 8. Both tables present out of sample results, involving the application of the models developed on the basis of the listed firms, to the sample of non-listed ones. The results are reported for each year separately, as well as overall (for the whole time-period). The best results under each classification setting (i.e., different PD thresholds) and year are marked in bold. Among the methods used in the comparison, ASVM provides the best results (overall) for both evaluation measures and under all classification settings. The AUROC, provide clearer conclusions on the relative performance of the different models, whereas the results of the KS distance are more mixed. Under AUROC, the ASVM results are slightly better with the full set of financial ratios (ASVM-all) compared to the ones with the reduced set of ratios selected through the stepwise LR procedure (ASVM-LR). Among the linear and RBF SVM models as well as LR, there are no striking differences. It is also worth noting that the results are rather robust for all methods under the three schemes used to perform the credit risk classification of the listed firms using different PD thresholds. Yet, the classification obtained by setting the PD threshold at 20%, seems to provide slightly better results overall (on average, however, the differences are very marginal).

Insert Tables 7 & 8 around here

3.4. Comparison to Models Fitted on Historical Default Data

In order to test the usefulness of the models constructed on the basis of the PD estimates obtained through the BSM model, a comparison was performed against credit rating models

(CRMs) developed using the historical default data available for the non-listed firms. Thus, in this case the CRMs were developed using as dependent variable the actual default status of the firms.

To construct these CRMs, the data over the period 2007–2008 were used for model fitting, whereas the 2009–2010 data were the holdout sample. Similarly to the approach used for the market models, stepwise LR is employed for the selection of the most important financial ratios. The results presented in Table 9 show that the stepwise procedure selected more variables compared to results for the listed firms (six vs. four). All variables in the LR model have the expected sign and so are the variables in the linear SVM models developed with both the reduced and the full set of variables (except for the S/STL ratio in the full LSVM model). As far as the contribution of the variables in the ASVM models is concerned, EBIT/TA is the most important ratio. This ratio was also found to be a strong predictor in the case of the market models analyzed in the previous section (cf. Table 6).

Insert Table 9 around here

Tables 10–11 present detailed comparative results on the predictive power of the market models analyzed in the previous section as opposed to the CRMs fitted on the actual default status of the non-listed firms. Only out-of-sample results are given involving the period 2009–2010. The best results (across all settings, i.e., CRM and the three market-based models) for each method and year are marked in bold. The results indicate that the market-based models are very competitive to the CRMs fitted on the actual default data. In particular, in overall terms (2009–2010) and under the AUROC criterion, the market-models developed with PD thresholds 20% and 30% are consistently very close to the CRMs. The two ASVM market models with a 20% PD threshold perform even better than the corresponding CRMs. Similar conclusions are also drawn with the KS criterion, where again the market-based models are found again to be very competitive to the CRM models.

Insert Table 10 & 11 around here

4. CONCLUSION AND FUTURE PERSPECTIVES

This study examined the development and implementation of a framework for building corporate credit scoring models based solely on publicly available data. To this end, the BSM structural model was used to introduce a proxy definition of default based on market data instead of the traditional approach based on the credit history of the firms. The market model's estimates of default were linked to models combining publicly available financial data. These models can be easily employed to any firm (listed or non-listed) in order to obtain estimates of its credit risk status.

The empirical application of this approach to data from Greece led to promising results. The obtained results demonstrated that, even in troublesome stock market conditions such as the ones that prevailed in the Greek stock market over the past decade, the predictability of the market-based models is very competitive to traditional credit rating models fitted on historical default data.

These positive preliminary results indicate that there is much room for future research that has the potential to provide many new capabilities and insights into credit risk modeling. A first obvious direction would be to employ a richer set of predictor attributes taking among others into account variables related to the business sector of the firms, variables related to non-financial characteristics of the firms (e.g., age, personnel, board member composition), corporate governance indicators, macroeconomic variables, as well as variables indicating the dynamics of the financial data of the firms (e.g., growth ratios). It is also necessary to examine the applicability of this modeling approach to developed international markets and consider the relationship of the results in comparison to credit ratings issued by major rating agencies. Finally, it is worth to investigate possible additional effects related to the recent debt crisis and other events that had significant impact on the international markets.

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Table 1: Number of observations in each sample

		Non-listed firms		
Years	Listed firms	Non-defaulted	Defaulted	Total
2005	192	–	–	–
2006	225	–	–	–
2007	225	2,748	52	2,800
2008	227	2,846	53	2,899
2009	224	2,731	99	2,830
2010	221	2,143	44	2,187
Total	1,314	10,468	248	10,716

Table 2: The selected financial ratios

Abbreviation	Ratios	Category	Expected sign
GP / S	Gross profit / Sales	Profitability	+
EBIT / TA	Earnings before taxes / Total assets	Profitability	+
TL / TA	Total liabilities / Total assets	Leverage	–
IE / S	Interest expenses / Sales	Solvency	–
CA / STL	Current assets / Short-term liabilities	Liquidity	+
S / STL	Sales / Short-term liabilities	Liquidity	+
AR / S	(Accounts receivable $\times$ 365) / Sales	Managerial performance	–

Table 3: Averages of the financial ratios for listed and non-listed firms

		Non-listed		
	Listed	Overall	Defaulted	Non-defaulted
GP / S	0.288	0.298	0.232	0.299
EBIT / TA	0.010	0.038	–0.040	0.040
TL / TA	0.603	0.719	0.879	0.716
IE / S	0.049	0.030	0.068	0.029
CA / STL	1.729	1.664	1.224	1.674
S / STL	2.394	2.547	1.509	2.572
AR / S	173.586	239.750	342.549	237.314

Table 4: Classification of listed firms and average PDs in each group

	PD threshold = 10%		PD threshold = 20%		PD threshold = 30%	
Years	Low risk	High risk	Low risk	High risk	Low risk	High risk
2005	163	29	182	10	191	1
	(1.12)	(17.41)	(2.39)	(25.25)	(3.41)	(36.09)
2006	207	18	212	13	218	7
	(0.91)	(25.70)	(1.23)	(30.00)	(1.85)	(35.58)
2007	190	35	203	22	207	18
	(1.35)	(41.68)	(2.16)	(58.03)	(2.61)	(65.27)
2008	137	90	191	36	217	10
	(2.95)	(19.99)	(6.35)	(27.50)	(8.50)	(35.89)
2009	142	82	173	51	204	20
	(1.84)	(24.28)	(4.15)	(30.08)	(7.36)	(37.54)
2010	92	129	128	93	160	61
	(2.03)	(28.79)	(5.75)	(34.03)	(9.59)	(38.80)
Total	931	383	1089	225	1197	117
	(1.59)	(25.93)	(3.49)	(33.81)	(5.41)	(42.19)

Table 5: Averages of the financial ratios for the risk groups defined from the market model (listed firms)

	PD threshold = 10%		PD threshold = 20%		PD threshold = 30%	
Ratios	Low risk	High risk	Low risk	High risk	Low risk	High risk
GP / S	0.306	0.242	0.300	0.228	0.293	0.233
IE / S	0.043	0.066	0.043	0.077	0.045	0.092
CA / STL	1.936	1.226	1.862	1.087	1.804	0.966
AR / S	157.872	211.785	159.810	240.261	165.116	260.244
EBIT / TA	0.034	–0.048	0.027	–0.070	0.019	–0.086
S / STL	2.725	1.589	2.605	1.369	2.517	1.130
TL / TA	0.544	0.746	0.562	0.803	0.578	0.859

Table 6: Contribution of variables in the linear and additive models developed using the market model's classifications

		GP/S	IE/S	CA/STL	AR/S	EBIT/TA	S/STL	TL/TA
10% PD threshold	LR	0.757	–	–	–	9.617	0.192	–4.074
	LSVM-all	0.080	0.182	–0.122	0.006	0.935	0.456	–1.058
	LSVM-LR	0.093	–	–	–	0.944	0.303	–0.931
	ASVM-all	0.050	0.183	0.169	0.053	0.206	0.201	0.137
	ASVM-LR	0.096				0.301	0.117	0.486
20% PD threshold	LR	1.240	–	–	–	8.992	0.219	–4.490
	LSVM-all	0.142	0.116	–0.006	0.071	0.811	0.662	–1.043
	LSVM-LR	0.156	–	–	–	0.884	0.471	–1.030
	ASVM-all	0.047	0.150	0.214	0.041	0.220	0.205	0.123
	ASVM-LR	0.115				0.234	0.117	0.534
30% PD threshold	LR	1.063	–	–	–	5.503	0.533	–5.453
	LSVM-all	0.158	0.146	–0.104	0.162	0.534	1.190	–1.230
	LSVM-LR	0.121	–	–	–	0.502	0.763	–1.194
	ASVM-all	0.040	0.117	0.245	0.068	0.190	0.190	0.149
	ASVM-LR	0.051				0.172	0.152	0.624

Table 7: Areas under the receiver operating characteristic curve for the sample of non-listed firms

PD thresh.	Methods	2007	2008	2009	2010	Overall
10%	LR	0.711	0.740	0.722	0.817	0.741
	LSVM-all	0.711	0.744	0.728	0.820	0.745
	LSVM-LR	0.718	0.752	0.739	0.829	0.754
	RBFSVM-all	0.704	0.738	0.738	0.825	0.747
	RBFSVM-LR	0.718	0.752	0.739	0.830	0.755
	ASVM-all	**0.735**	**0.773**	**0.756**	0.830	**0.771**
	ASVM-LR	0.733	0.769	0.750	**0.849**	0.770
20%	LR	0.714	0.746	0.731	0.827	0.748
	LSVM-all	0.713	0.747	0.732	0.828	0.749
	LSVM-LR	0.719	0.753	0.740	0.830	0.755
	RBFSVM-all	0.722	0.749	0.743	0.833	0.757
	RBFSVM-LR	0.680	0.752	0.719	0.812	0.736
	ASVM-all	**0.763**	**0.791**	**0.770**	**0.852**	**0.790**
	ASVM-LR	0.749	0.780	0.765	0.843	0.781
30%	LR	0.708	0.748	0.737	0.831	0.751
	LSVM-all	0.703	0.739	0.730	0.823	0.743
	LSVM-LR	0.714	0.753	0.737	0.821	0.752
	RBFSVM-all	0.696	0.743	0.735	0.813	0.743
	RBFSVM-LR	0.721	0.753	0.748	0.833	0.760
	ASVM-all	**0.759**	**0.790**	**0.765**	**0.851**	**0.787**
	ASVM-LR	0.730	0.770	0.757	0.838	0.770

Table 8: Kolmogorov-Smirnov distances for the sample of non-listed firms

PD thresh.	Methods	2007	2008	2009	2010	Overall
10%	LR	0.403	0.421	0.380	0.511	0.389
	LSVM-all	0.402	0.430	0.396	0.494	0.402
	LSVM-LR	0.410	0.461	0.404	0.541	0.418
	RBFSVM-all	0.329	0.407	0.401	0.557	0.396
	RBFSVM-LR	**0.411**	0.469	0.399	0.542	0.417
	ASVM-all	0.399	0.419	**0.411**	0.569	**0.424**
	ASVM-LR	**0.411**	**0.495**	0.397	**0.584**	0.431
20%	LR	0.392	0.452	0.394	0.521	0.412
	LSVM-all	0.386	0.436	0.396	0.518	0.407
	LSVM-LR	0.411	0.462	0.401	0.538	0.418
	RBFSVM-all	0.392	0.446	0.396	0.569	0.427
	RBFSVM-LR	0.333	**0.507**	0.389	0.513	0.404
	ASVM-all	**0.428**	0.463	**0.431**	0.582	0.440
	ASVM-LR	0.424	0.480	0.423	**0.584**	**0.459**
30%	LR	0.374	0.439	0.425	0.521	0.408
	LSVM-all	0.363	0.431	0.418	0.527	0.402
	LSVM-LR	0.379	0.431	0.429	0.496	0.407
	RBFSVM-all	0.346	0.405	0.385	0.510	0.393
	RBFSVM-LR	0.359	0.450	0.403	0.555	0.418
	ASVM-all	**0.427**	0.462	0.423	**0.600**	**0.448**
	ASVM-LR	0.394	**0.467**	**0.443**	0.585	0.440

Table 9: Contribution of variables in the linear and additive models developed using the sample of non-listed firms

Ratios	LR	LSVM-all	LSVM-LR	ASVM-all	ASVM-LR
GP / S	0.981	0.155	0.177	0.121	0.147
IE / S	–8.521	–0.442	–0.396	0.127	0.154
CA / STL	0.213	0.149	0.089	0.163	0.198
AR / S	–0.001	–0.293	–0.243	0.091	0.110
EBIT / TA	2.290	0.298	0.305	0.202	0.246
S / STL	–	–0.134	–	0.177	
TL / TA	–1.349	–0.459	–0.458	0.119	0.144

Table 10: Comparison of the market models' results to credit risk models developed for non-listed firms (areas under the receiver operating characteristic curve)

		LR	LSVM-all	LSVM-LR	RBFSVM-all	RBFSVM-LR	ASVM-all	ASVM-LR
2009	CRM	**0.760**	**0.759**	**0.762**	**0.760**	**0.761**	0.767	0.759
	10%	0.722	0.728	0.739	0.738	0.739	0.756	0.750
	20%	0.731	0.732	0.740	0.743	0.719	**0.770**	**0.765**
	30%	0.737	0.730	0.737	0.735	0.748	0.765	0.757
2010	CRM	0.802	0.800	0.809	0.818	0.818	0.839	0.843
	10%	0.817	0.820	0.829	0.825	0.830	0.830	**0.849**
	20%	0.827	**0.828**	**0.830**	**0.833**	0.812	**0.852**	0.843
	30%	**0.831**	0.823	0.821	0.813	**0.833**	0.851	0.838
Overall	CRM	**0.773**	**0.772**	**0.777**	**0.778**	**0.779**	0.786	**0.789**
	10%	0.751	0.756	0.766	0.765	0.767	0.780	0.780
	20%	0.761	0.763	0.768	0.771	0.748	**0.796**	**0.789**
	30%	0.767	0.760	0.764	0.760	0.775	0.792	0.783

Table 11: Comparison of the market models' results to credit risk models developed for non-listed firms (Kolmogorov-Smirnov distances)

		LR	LSVM-all	LSVM-LR	RBFSVM-all	RBFSVM-LR	ASVM-all	ASVM-LR
2009	CRM	0.412	0.411	0.423	**0.407**	**0.410**	**0.433**	0.413
	10%	0.380	0.396	0.404	0.401	0.399	0.411	0.397
	20%	0.394	0.396	0.401	0.396	0.389	0.431	0.423
	30%	**0.425**	**0.418**	**0.429**	0.385	0.403	0.423	**0.443**
2010	CRM	**0.522**	**0.531**	0.535	0.529	0.507	0.561	0.576
	10%	0.511	0.494	0.541	0.557	0.542	0.569	0.584
	20%	0.521	0.518	**0.538**	**0.569**	0.513	0.582	0.584
	30%	0.521	0.527	0.496	0.510	**0.555**	**0.600**	**0.585**
Overall	CRM	**0.445**	**0.439**	**0.453**	**0.442**	**0.441**	0.460	0.462
	10%	0.410	0.416	0.433	0.441	0.430	0.447	0.439
	20%	0.419	0.424	0.429	**0.442**	0.421	**0.474**	**0.466**
	30%	0.443	0.431	0.439	0.420	0.435	0.464	0.464

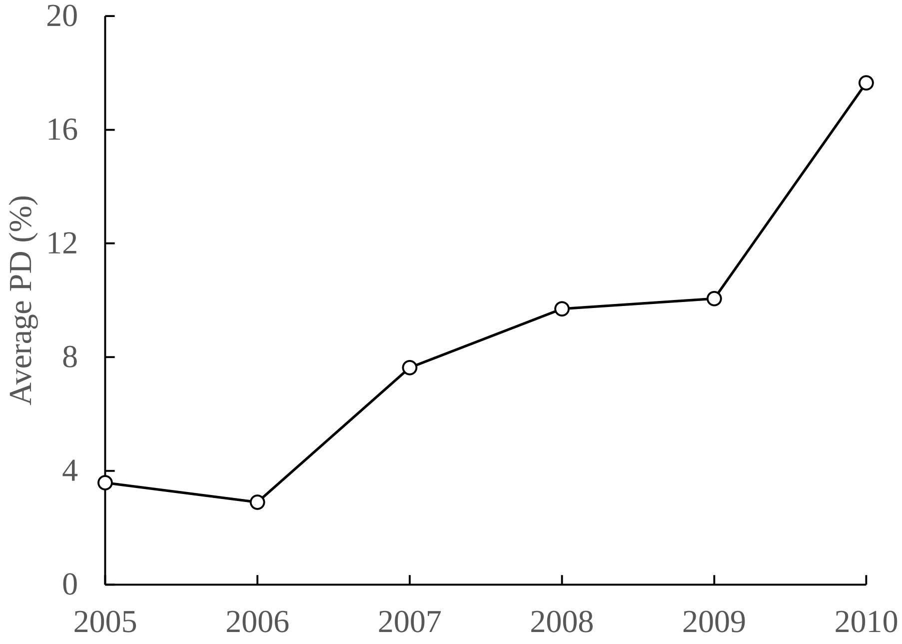

Figure 1: Average probability of default for listed firms according to the market model

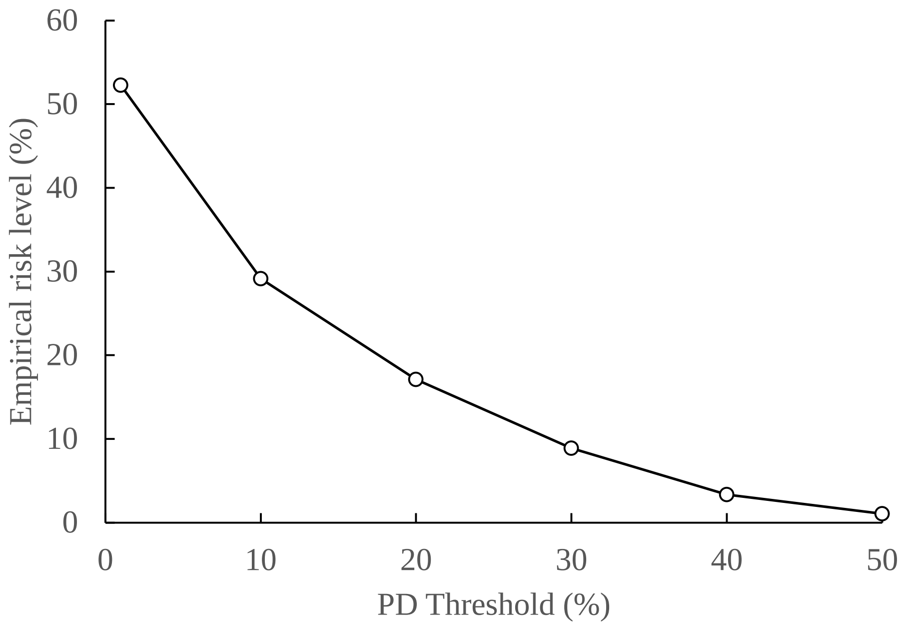

Figure 2: Relationship between the PD threshold and the empirical risk level

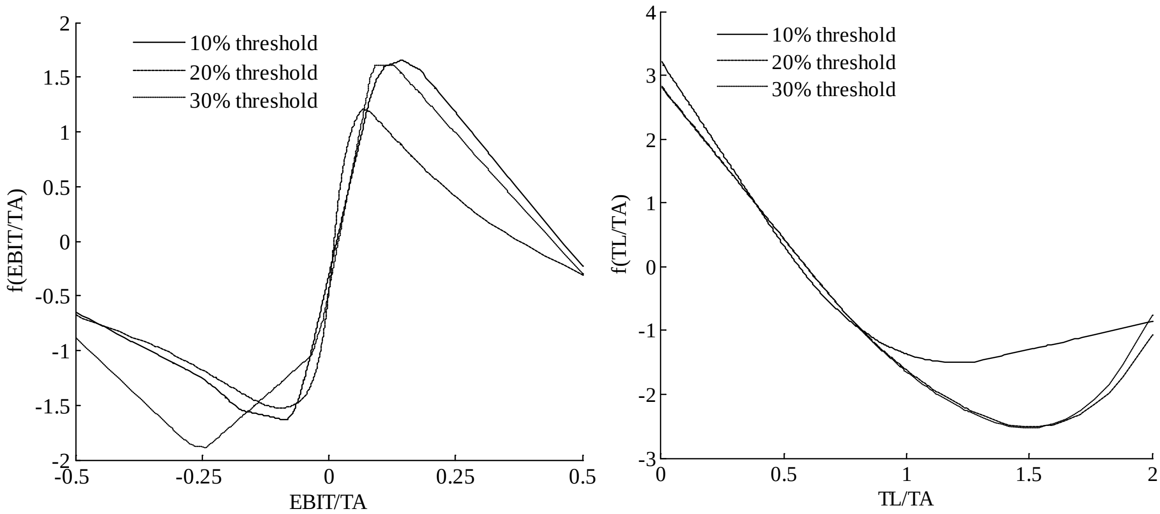

Figure 3: Partial evaluation functions of EBIT/TA and TL/TA ratios